Lesson 8-1 The Integral as Net Change

AP Calculus AB
Lessons 8-1: *The Integral as Net Change*

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Date _____

Learning Goal:

- *I can solve problems in which rate is integrated to find the net change over time in a variety of applications.*

*** **For all problems, use your calculator to evaluate integrals unless the problem specifies to evaluate the integral analytically.** ***

Displacement – the amount of movement of an object measured in a particular direction; measured by the final distance from the starting point; for example suppose on a horizontal axis a particle starting at $x=1$ travels from $x=1$ to $x=6$ and then from $x=6$ back to $x=3$ and stops at $x=3$. The particle would have a displacement of 2 units because it started at $x=1$ and ended at $x=3$.

Position – where an object is relative to "zero". The position of the particle in the above example at the end of its motion is 3, because it ends at $x=3$. Note that the final position of the particle is its starting position + its displacement.

Total Distance Traveled – total amount of distance moved by the object. Using the above example in the displacement definition, the total distance traveled would be 8 units because the particle started at $x=1$ and moved to $x=6$ (traveling 5 units) and then back to $x=3$ (traveling 3 units) for a total of $5+3=8$ units)

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1. A particle moving along a horizontal s-axis has velocity given by $\dfrac{ds}{dt}=v(t)=t^2-\dfrac{8}{(t+1)^2}$ cm/sec for

   $[0,5]$.

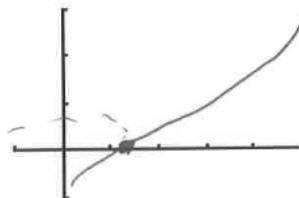   a. What is the initial velocity of the particle?
   Graph the velocity function and use the graph to describe its motion at $t=0$. $[0,5] \times [-10, 30]$

   $v(0) = 0 - \dfrac{8}{1} = -8$

   Description
   The particle is moving left at 8 cm/sec

   b. Use your calculator to find when is the particle at rest?
   $t = 1.255$ sec

   c. When is the particle moving left? right?

   $v(t)$ under s-axis $v(t)$ above s-axis
   The particle is moving left from $[0, 1.255)$ and right from $(1.255, 5]$. OVER →

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d. Find the **displacement** for the particle for t on [0, 5]:

i. analytically: displacement $= \int_0^5 \left(t^2 - \dfrac{8}{(t+1)^2} \right) dt$

$$\int_0^5 t^2 - \int_0^5 \frac{8}{(t+1)^2}$$

$u = t+1$

$\dfrac{du}{dt} = 1$

$du = dt$

$\int \dfrac{8}{u^2} \, du$

$= -\dfrac{8}{u}$

$$= \frac{1}{3} t^3 \Big|_0^5 \; - -\frac{8}{(t+1)} \Big|_0^5$$

$$= \frac{(5)^3}{3} - 0 + \frac{8}{6} - \frac{8}{1}$$

$$= \frac{125}{3} + \left(\frac{8}{6} - \frac{8}{1} \right) =$$

$$= \frac{125}{3} - \frac{40}{6} = \frac{210}{6} = \boxed{35 \, cm}$$

ii. numerically (on your calculator): $\int_0^5 v(t) \, dt = \boxed{35 \, cm}$

e. Find the **total distance traveled** by the particle for t on [0, 5].

i. Distanced traveled $= \int_0^5 |v(t)| \, dt = 42.587 \, sec$

(use calculator)

Neutralize negative

ii. How would you find this answer analytically? Set-up the needed integral(s) but do not evaluate them.

Integrate: $\left| \int_0^{1.255} v(t) \right| + \int_{1.255}^{5} v(t)$

below
s-axis
abs. value
neutralizes
the
negative.

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f. Suppose the initial position of the particle is s(0) = 9. Find s(1) and s(5). *Use the FTC!*
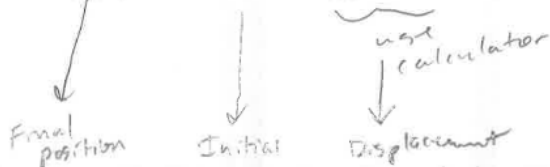
$$v(t) = t^2 - \frac{8}{(t+1)^2}$$

$$\int_0^1 v(t) = S(1) - S(0)$$

$$S(1) = S(0) + \int_0^1 v(t)$$

$$= 9 + \int_0^1 v(t) = \boxed{5.333}$$

use
calculator.

$$\int_0^5 v(t) = S(5) - S(0)$$

$$S(5) = S(0) + \int_0^5 v(t) = \boxed{44}$$

use
calculator

Final   Initial   Displacement
position

***Note that the rearranged FTC always can be written as *final position = initial position + displacement*

g. Find the position of the particle at the following times.

   i. t = 2: $9 + \int_0^2 v(t) = \boxed{6.333}$

   ii. t = 4: $9 + \int_0^4 v(t) = \boxed{23.933}$

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.2. A car moving with a velocity of 5 mph accelerates at the rate of $a(t) = 2.4t$ mph per second for 8 seconds. How fast is the car going when the 8 seconds are up?

$$\int_0^8 a(t) = V(8) - V(0)$$

$$V(8) = V(0) + \int_0^8 a(t)$$

$$= 5 + \int_0^8 a(t)$$

$$= 81.8 \text{ mph}$$

3. A pump connected to a generator operates at a varying rate, depending on how much power is being drawn from the generator to operate other machinery. The rate (gallons per minute) at which the pump operates is recorded at 5-minute intervals for one hour as shown in the table at the right. How many gallons were pumped during that hour? Use what you believe is the most accurate method to solve this problem.

Time (min)	Rate (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

Gallons pumped $= \int_0^{60} R(t) dt$

Since we only have data for $R(t)$, we can use trapezoidal method to estimate integral. Using the intervals given in the table:

$$\int_0^{60} R(t) dt = \frac{1}{2} \cdot 5 \left((58+60) + (60+65) + \ldots, (63+63) \right)$$

$$= 3582.5 \text{ gallons}$$

·4. If $a(t)$ represents the acceleration of the water being moved by the pump, find $\int_{15}^{45} a(t) dt$. Explain what your answer means in the context of the problem.

$$\int_{15}^{45} a(t) dt = R(45) - R(15) = 60 \text{ gal/min} - 64 \text{ gal/min}$$

$$= -4 \text{ gal/min}$$

The velocity decreased by 4 gal/min over the interval $t=15$ to $t=45$.